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ABSTRACT

This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. The content of this package centers on work with the ratio of two polynomials (fractions). Techniques for manipulating and simplifying algebraic fractions are presented. This work was prepared under an ESEA Title III contract. (JP)

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ED 070 019

ALGEBRA I

PACKAGE 03-08

OPERATIONS WITH FRACTIONS

Prepared by

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SE 017 572

Many problems and equations require the use of fractions in their solution. In this package you will learn how to add, subtract, multiply, and divide fractions containing polynomials in their numerators and denominators. You will also learn what a ratio is and how to use ratios to help you solve problems. You will find this part of algebra very similar to arithmetic.

The goal of this package is:

for you to learn how to perform the fundamental operations with fractions and for you to learn how to write and use ratios to help solve problems.

PACKAGE OBJECTIVES:

1. Given an expression in the form of a fraction, write the values of the variable (or variables) for which the fraction is not defined.
2. Given a fraction, write it in lowest terms.
3. Given a problem involving the use of ratios, solve it.
4. Given a problem which requires application of the percentage formula, solve it.
5. Given two fractions, write their product in lowest terms.
6. Given two fractions, write their quotient in lowest terms.
7. Given a series of fractions to be multiplied and divided, perform the indicated operations and write the answer in lowest terms.
8. Given two fractions with equal denominators write their sum or difference as a simple fraction in lowest terms.
9. Given two fractions with different denominators write their sum or difference in lowest terms.
10. Given a mixed expression, write it as a fraction in lowest terms, and conversely.
11. Given a complex fraction, simplify it.

I. U. #03-08-01

DEFINING FRACTIONS

OBJECTIVES:

1. When asked to define a fraction write, "A fraction is a numeral of the form $\frac{a}{b}$ where $b \neq 0$.

2. Given an expression which is not in the form of a fraction, write it in fractional form.

3. Given an expression in the form of a fraction write the values of the variable (or variables) for which the fraction is undefined.

ACTIVITIES:

1. Study objective one until you understand the definition of a fraction and can repeat it from memory. (Objective 1)

2. Using the definition, express each odd numbered part A exercise on page 296, SM, as a fraction. (Objective 2)

3. Study page 295 and state any values of the variable for which the fraction is not defined in the odd numbered part A exercises on page 296. (Objective 3)

4. You should try a few part B exercises and if you can get some of the part C exercises, congratulations!

Criterion Test 03-08-01-01

1. Define a fraction.

2. Write as a fraction.

   (A) $5 \div x$

   (B) .23

   (C) $(x - y) \div (a + b)$

   (D) $5 \div (m + n)$

   (E) $(5x^2 - 3) \div 2$

3. Write the values of the variables for which the fraction is undefined.

   (A) $\frac{6}{x}$ (B) $\frac{75}{10T - 30}$

   (C) $\frac{13}{x(x - 5)}$ (D) $\frac{x - 1}{x}$

Criterion Test 03-08-01-02

1. Define a fraction.

2. Write as a fraction.

   (A) $3 \div m$

   (B) .273

   (C) $(3x + 5) \div (x - 2)$

   (D) $13 \div (2m + n)$

   (E) $(3ab - b^2) \div 5$

3. Write the values of the variables for which the fraction is undefined.

   (A) $\frac{15}{a}$

   (B) $\frac{35}{5T - 35}$

   (C) $\frac{1}{m(m - 17)}$

   (D) $\frac{3x^2 + 2x + 1}{x}$

Criterion Test 03-08-01-03

1. Define a fraction.

2. Write as a fraction.

   (A) $x \div 6$

   (B) .15

   (C) $(13 + x) \div (x - 2)$

   (D) $5 \div (x - 1)$

   (E) $(x - 1) \div 5$

3. Write the values of the variables for which the fraction is undefined.

   (A) $\frac{25}{z}$

   (B) $\frac{25}{3z - 12}$

   (C) $\frac{25}{z(z - 25)}$

   (D) $\frac{(z - 25)}{z}$

Answers to Criterion Tests

Test 03-08-01-01

1. A fraction is a numeral of the form $\frac{a}{b}$ where $b \neq 0$.

2. (A) $\frac{5}{x}$ (B) $\frac{23}{100}$ (C) $\frac{x - y}{a + b}$ (D) $\frac{5}{m + n}$

(E) $\frac{5x^2 - 3}{2}$

3. (A) $x = \{0\}$ (B) $T = \{3\}$ (C) $x = \{5, 0\}$ (D) $x = \{0\}$

Test 03-08-01-02

1. A fraction is a numeral of the form $\frac{a}{b}$ where $b \neq 0$.

2. (A) $\frac{3}{m}$ (B) $\frac{273}{1000}$ (C) $\frac{3x + 5}{x - 2}$ (D) $\frac{13}{2m + n}$

3. (A) $a = \{0\}$ (B) $T = \{7\}$ (C) $m = \{17, 0\}$ (D) $x = \{0\}$

Test 03-08-01-03

1. A fraction is a numeral of the form $\frac{a}{b}$ where $b \neq 0$

2. (A) $\frac{x}{6}$ (B) $\frac{15}{100}$ or $\frac{3}{20}$ (C) $\frac{13 + x}{x - 2}$

   (D) $\frac{5}{x - 1}$ (E) $\frac{x - 1}{5}$

3. (A) $z = \{0\}$ (B) $z = \{4\}$ (C) $z = \{25, 0\}$

   (D) $\{z = 0\}$

I. U. #03-08-02

REDUCING FRACTIONS TO LOWEST TERMS

OBJECTIVES:

1. When asked to write the multiplication property of fractions, write " $\frac{ac}{bc} = \frac{a}{b}$ provided $c \neq 0$.

2. Given a fraction, write it in lowest terms.

ACTIVITIES:

1. Study the multiplication property of fractions on page 297, SM, until you understand it and can repeat it in mathematical symbols from memory.
(Objective 1)

2. Review the meaning of "Greatest common factor", if necessary, (page 250 SM), then study the rest of page 297, SM, until you understand how to reduce a fraction to lowest terms.
(Objective 2)

3. Do the odd numbered part A exercises on page 298.
(Objective 2)

   You should be able to do some of the part B exercises if you have mastered all of the work to this point. Try some part C exercises if you want to give yourself a more challenging test.

Criterion Test 03-08-02-01

1. Write the multiplication property of fractions.

2. Write in lowest terms.

(A) $\frac{28b^3}{b^2}$

(B) $\frac{13x - 13y}{x^2 - y^2}$

(C) $\frac{7x^2 - 28}{x + 2}$

(D) $\frac{x^2 - 9}{x^2 + 6x + 9}$

(E) $\frac{(x - 3)^2}{x^2 - 9}$

Criterion Test 03-08-02-02

1. Write the multiplication property of fractions.

2. Write in lowest terms.

(A) $\frac{35x^2}{7x}$ (B) $\frac{4a + 4b}{a^2 - b^2}$

(C) $\frac{3x^2 - 12}{x + 2}$ (D) $\frac{x^2 - 16}{x^2 - 8x + 16}$

(E) $\frac{x^2 + 8x + 16}{2x + 8}$

Criterion Test 03-08-02-03

1. Write the multiplication property of fractions.

2. Write in lowest terms.

(A) $\frac{24x^2}{8}$ (B) $\frac{m^2 - n^2}{m + n}$

(C) $\frac{m - n}{m^2 - n^2}$ (D) $\frac{x^2 - 25}{x^2 + 6x + 5}$

(E) $\frac{x^2 + 4x + 3}{x^2 + 5x + 6}$

Answers to Criterion Tests

Test 03-08-02-01

1. $\frac{ac}{bc} = \frac{a}{b}$ provided $c \neq 0$.

2. (A) $28b$ (B) $\frac{13}{x + y}$ (C) $7(x - 2)$

(D) $\frac{x - 3}{x + 3}$ (E) $\frac{x - 3}{x + 3}$

Test 03-08-02-02

1. $\frac{ac}{bc} = \frac{a}{b}$ provided $c \neq 0$.

2. (A) $5x$ (B) $\frac{4}{a - b}$ (C) $3(x - 2)$

(D) $\frac{x + 4}{x - 4}$ (E) $\frac{x + 4}{2}$

Test 03-08-02-03

1. $\frac{ac}{bc} = \frac{a}{b}$ provided $c \neq 0$.

2. (A) $3x^2$ (B) $m - n$ (C) $\frac{1}{m + n}$

(D) $\frac{x - 5}{x + 1}$ (E) $\frac{x + 1}{x + 2}$

I. U. #03-08-03

RATIO

OBJECTIVES:

1. When asked to define ratio write "A ratio of one number to another one (not zero) is the quotient of the first number divided by the second."

2. Given a ratio, express it in four different ways.

3. Given two quantities of the same kind, write their ratio in lowest terms.

4. Given a problem involving the use of ratios, solve it.

ACTIVITIES:

1. Study objective 1 until you can define ratio from memory.
   (Objective 1)

2. Study page 299 until you are sure you can express a ratio in four different ways.
   (Objective 2)

3. Study page 300 and do enough of the part A exercises on page 301 and the part A problems on page 302 to be sure that you have met objective 3. You will find the part "B" exercises and problems more challenging and therefore especially rewarding to do.

Criterion Test 03-08-03-01

1. Define ratio.

2. Write the ratio in lowest terms.

   (A) The area of a 3 inch by 4 inch rectangle to the area of a square 2 inches on a side.

   (B) The ratio of boys to girls in a school with 500 students and 200 girls.

   (C) Wins to losses if a team has won five out of ten games played.

3. Solve:

   (A) If a car goes 52 miles on 4 gallons of gas , how far will it go on 10 gallons?

   (B) Miss Able types 2000 words in 50 minutes while Miss Stake types, 738 words in 19 minutes. Which is the faster typist?

   (C) If an alloy contains five parts of carbon to 95 parts of iron, by weight, how many pounds of iron are in a ton of the alloy?

Criterion Test 03-08-03-02

1. Define ratio.

2. Write the ratio in lowest terms.

   (A) The area of a 6 inch square to the area of a 2 inch square.

   (B) The perimeter of a six inch square to the perimeter of a 2 inch square.

   (C) A side of a six inch square to a side of a two inch square.

3. Solve:

   (A) Find the lesser of two numbers in the ratio of 3 to 7 whose difference is 8.

   (B) Find the lesser of two numbers whose ratio is 3 to 7 and whose sum is 20.

   (C) If a car goes 42 miles on 3 gallons of gas, how far will it go on 10 gallons of gas.

Criterion Test 03-08-03-03

1. Define ratio.

2. Write the ratio in lowest terms.

   (A) The ratio of cost per ounce for margarine that costs 64¢ per pound.

   (B) The ratio of miles per hour on a 496 mile trip that takes 8 hours.

   (C) The ratio of 328 hits to 350 attempts at a shooting contest.

3. Solve:

   (A) If the ratio of saleable meat to weight of a steer is .65, how many pounds of saleable meat is there on a 1000 pound steer?

   (B) If there is 55 pounds of saleable meat per 100 pounds of live steer, how much must a steer weigh to yield 500 pounds of saleable meat?

   (C) Which is a better buy, a 16 ounce can at 70¢ or a 12 ounce can at 57¢?

Answers to Criterion Tests

Test 03-08-03-01

1. A ratio of one number to another one (not zero) is the quotient of the first number divided by the second.

2. (A) $\frac{12}{4}$ or $\frac{3}{1}$ (B) $\frac{300}{200}$ or $\frac{3}{2}$ or $\frac{1.3}{1}$

   (C) $\frac{5}{5}$ or $\frac{1}{1}$

3. (A) 130 miles (B) Able 40 words per minute
   Stake 38.3 words per minute
   Able is the fastest.

   (C) 1900 pounds

Test 03-08-03-02

1. A ratio of one number to another (not zero) is the quotient of the first number divided by the second.

2. $\frac{36}{4}$ or $\frac{9}{1}$ (B) $\frac{24}{8}$ or $\frac{3}{1}$ (C) $\frac{4}{2}$ or $\frac{3}{1}$

3. (A) 6 (B) 6 (C) 140 miles

Answers to Criterion Test (Cont.)

Test 03-08-03-03

1. A ratio of one number to another (not zero) is the quotient of the first number divided by the second.

2. (A) 4¢ per ounce or $\frac{4}{1}$

   (B) 62 miles per hour or $\frac{62}{1}$

   (C) $\frac{164}{175}$ or .9371

3. (A) 650 pounds (B) $909.\overline{09}$ pounds

   (C) The 16 ounce can at 4.37¢ per ounce is better than the 12 ounce can at 4.75¢ per ounce.

I. U. #03-08-04

PERCENT AND PERCENTAGE PROBLEMS

OBJECTIVES:

1. Given a fraction, write it as a percent.

2. Given a decimal, write it as a percent.

3. When asked to write the percentage formula write,

   $p = rb$ and $\frac{p}{b} = r \quad b \neq 0$.

4. Given a problem which requires application of the percentage formula, solve it.

ACTIVITIES:

1. Study pages 303, 304 in SM.
   (Objectives 1, 2, 3)

2. Write enough of the odd numbered exercises on pages 304 and 305 to be sure that you have achieved objective 4.

3. Do the odd numbered part A problems on page 305. If you like a challenge try some of the part "B" problems.

Criterion Test 03-08-04-01

1. Write as a percent.

   (A) $\frac{5}{8}$ (B) $\frac{1}{6}$

2. Write as a percent.

   (A) 2.53 (B) .05

3. Write the percentage formula.

4. Solve the following problems.

   (A) What is 12% of 180?

   (B) 18 is 12% of what number?

   (C) What % of 35 is 30?

   (D) The price of a radio is $45.00. It was sold at a discound of $5.00. What is the percent of discount?

   (E) If 30% of 150 students are freshmen, how many are freshmen?

Criterion Test 03-08-04-02

1. Write as a percent.

   (A) $\frac{7}{8}$ (B) $\frac{1}{7}$

2. Write as a percent.

   (A) .5 (B) 1.25

3. Write the percentage formula.

4. Solve the following problems.

   (A) What is 3% of 4.5?

   (B) What % of 4.5 is 3?

   (C) 4.5 is 3% of what number?

   (D) Rod received 75 school votes for president. If that was 55% of them, how many school votes are there?

   (E) If the bank charges 7½% interest on $345.00, how much is the interest for one year?

Criterion Test 03-08-04-03

1. Write as a percent.

   (A) $\frac{7}{4}$ (B) $\frac{1}{9}$

2. Write as a percent.

   (A) .342 (B) 2.75

3. Write the percentage formula.

4. Solve the following problems:

   (A) What is the number that is 7% of 45?

   (B) What is the number that is 45% of 7?

   (C) What % of 45 is 7?

   (D) If a car got 15 miles per gallon of gas and improved its mileage by 15% by putting in new spark plugs, how much was the increase?

   (E) If the church has collected $35,000.00 and that is 20% of the cost of the new church, how much will the new church cost?

Answers to Criterion Tests

Test 03-08-04-01

1. (A) 62.5% (B) $16\frac{2}{3}\%$ or $16.\overline{6}\%$

2. (A) 253% (B) 5%

3. $p = rb$ or $\frac{p}{b} = r$ $b \neq 0$

4. (A) 21.6 (B) 150 (C) 85.7%

   (D) $11.\overline{1}\%$ discount (E) 45 students are freshmen

Test 03-08-04-02

1. (A) 87.5% (B) 14.28% or 14.3%

2. (A) 50% (B) 125%

3. $p = rb$ or $\frac{p}{b} = r$ $b \neq 0$

4. (A) .135 (B) $66.\overline{6}\%$ (C) 150

   (D) 136 students or 137 students (75 is 55% of either of these two numbers. Remember we cannot have a fraction of a student in this problem.)

   (E) $25.88 (Remember, to round half a cent to the next cent.)

Answers to Criterion Tests (Cont.)

Test 03-08-04-03

1. (A) 175% (B) $11.\overline{1}\%$

2. (A) 34.2% (B) 275%

3. $p = rb$ or $\frac{p}{b} = r \quad b \neq 0$

4. (A) 3.15

   (B) 3.15

   (C) $15.\overline{5}\%$

   (D) 2.25 miles per gallon or $2\frac{1}{4}$ mile per gallon

   (E) $175,000.00

I. U. #03-08-05

MULTIPLYING FRACTIONS

OBJECTIVES:

1. When asked to write the rule for multiplying fractions write, "For any real numbers x, y, c, and d, if $c \neq 0$ and $d \neq 0$, then $\frac{x}{c} \cdot \frac{y}{d} = \frac{xy}{cd}$

2. Given two fractions, write their product in lowest terms.

ACTIVITIES:

1. Study pages 307 and 308 in SM.
   (Objectives 1, 2)

2. Write the odd numbered part A problems on page 309 and try some of the part B problems. For a real challenge do a few part C problems.

Criterion Test 03-08-05-01

1. Write the rule for multiplying fractions.
2. Write the product in lowest terms.

(A) $\frac{1}{3} \cdot \frac{2}{5} \cdot 4$ (B) $\frac{2}{3} \cdot \frac{4}{5} \cdot (-2)$

(C) $6a^2b \cdot \frac{2}{3a^2}$ (D) $\frac{2T + 14}{6T} \cdot \frac{9T^2}{T^2 + 7T}$

Criterion Test 03-08-05-02

1. Write the rule for multiplying fractions.
2. Write the product in lowest terms.

(A) $\frac{1}{2} \cdot \frac{2}{3} \cdot 5$ (B) $\frac{1}{3} \cdot \frac{2}{5} \cdot -4$

(C) $ab^2 \cdot \frac{a}{2b^2 + 3b}$ (D) $\frac{x^2 - y^2}{x - y} \cdot \frac{x^2 - 2xy + y^2}{x - y}$

Criterion Test 03-08-05-03

1. Write the rule for multiplying fractions.
2. Write the product in lowest terms.

(A) $\frac{2}{5} \cdot 3 \cdot \frac{4}{7}$ (B) $\frac{5}{2} \cdot (-2) \cdot \frac{7}{3}$

(C) $3ab \cdot \frac{a + b}{ab^2}$ (D) $\frac{3x - 2y}{xy} \cdot \frac{3xy}{9x^2 - 12xy + 4y^2}$

Answers to Criterion Tests

Test 03-08-05-01

1. For any real numbers x, y, c, and d, if $c \neq 0$ and $d \neq 0$, then $\frac{x}{c} \cdot \frac{y}{d} = \frac{xy}{cd}$

2. (A) $\frac{8}{15}$

   (B) $-\frac{16}{15}$

   (C) 4b

   (D) 3

Test 03-08-05-02

1. For any real numbers x, y, c, and d, if $c \neq 0$ and $d \neq 0$, then $\frac{x}{c} \cdot \frac{y}{d} \cdot \frac{xy}{cd}$

2. (A) $\frac{5}{3}$ or $1\frac{2}{3}$

   (B) $-\frac{8}{15}$

   (C) $\frac{a^2b}{2b + 3}$

   (D) $x^2 - y^2$

Answers to Criterion Tests (Cont.)

Test 03-08-05-03

1. For any real numbers x, y, c, and d, if $c \neq 0$ and $d \neq 0$, then $\frac{x}{c} \cdot \frac{y}{d} = \frac{xy}{cd}$

2. (A) $\frac{24}{35}$

   (B) $\frac{-35}{3}$

   (C) $\frac{3(a + b)}{b}$

   (D) $\frac{3}{3x - 2y}$

I. U. #03-08-06

DIVIDING FRACTIONS

OBJECTIVES:

1. When asked to write the rule for dividing fractions write, "For any real numbers a, b, c, and d, if $b \neq 0$ $c \neq 0$, and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

2. Given two fractions, write their quotient in lowest terms.

ACTIVITIES:

1. Study page 370 SM.
   (Objectives 1, 2)

2. If you have trouble understanding why $4 \div \frac{1}{2} = 8$ study the diagram on page 396 of "Algebra One".

3. Do enough of the odd numbered part A exercises on page 311 of SM to be sure you have achieved objective 2. Part B and C are especially rewarding to work. (The reward is the satisfaction of being able to work a more difficult exercise correctly.)

Criterion Test 03-08-06-01

1. Write the rule for dividing fractions.
2. Write the quotient in lowest terms.

(A) $\frac{3}{5} \div \frac{6}{15}$

(B) $\frac{a}{b^2} \div \frac{a^2}{b}$

(C) $\frac{a^2 - 4}{x} \div a + 2$

(D) $\frac{y^2 - 9}{y^2 - 6y + 9}$

Criterion Test 03-08-06-02

1. Write the rule for dividing fractions.
2. Write the quotient in lowest terms.

(A) $\frac{2}{3} \div \frac{3}{7}$

(B) $\frac{a^2}{x} \div \frac{3ax}{4}$

(C) $\frac{x^2 - y^2}{x} \div \frac{x^2 - 2xy + y^2}{x}$

(D) $\frac{x^2 - 9}{x} \div \frac{3x - 9}{4x}$

Criterion Test 03-08-06-03

1. Write the rule for dividing fractions.

2. Write the quotient in lowest terms.

(A) $\frac{3}{7} \div \frac{6}{5}$

(B) $\frac{ab}{c} \div \frac{c^2}{ab}$

(C) $\frac{1 - x^4}{x} \quad \frac{x^2 - 4x + 4}{x^2}$

(D) $\frac{x^2 - 4}{x^3} \quad \frac{x^2 - 4x + 4}{x^2}$

Answers to Criterion Tests

Test 03-08-06-01

1. For any real numbers a, b, c, and d, if $b \neq 0$, $c \neq 0$ and $d \neq 0$ then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

2. (A) $\frac{3}{2}$ (B) $\frac{1}{ab}$ (C) $\frac{a - 2}{x}$ (D) $\frac{y + 3}{y - 3}$

Test 03-08-06-02

1. For any real numbers a, b, c, and d, if $b \neq 0$, $c \neq 0$, and $d \neq 0$ then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

2. (A) $\frac{14}{9}$ or $1\frac{5}{9}$ (B) $\frac{4a}{3x^2}$

(C) $\frac{x + y}{x - y}$ (D) $\frac{4(x + 3)}{3}$

Test 03-08-06-03

1. For any real numbers a, b, c, and d, if $b \neq 0$, $c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

2. (A) ~~$\frac{5}{21}$~~ $\frac{3}{14}$ (B) $\frac{a^2b^2}{c^3}$

(C) $\frac{x}{1 + x^2}$ (D) $\frac{x + 2}{x(x - 2)}$

I. U. #03-08-07

EXPRESSIONS

INVOLVING

MULTIPLICATION AND DIVISION

You will need to recall:

The rule for order of performing multiplications and divisions.

OBJECTIVES:

1. Given a series of fractions to be multiplied and divided, perform the indicated operations and write the answer in lowest terms.

ACTIVITIES:

1. If necessary, review pages 41 and 42, SM, to recall the order of doing operations in an expression. (Objective 1)

2. Study page 312, SM, and do the odd numbered part A exercises. (Try part B for a greater challenge.) (Objective 1)

Criterion Test 03-08-07-01

1. Perform the indicated operations and write the answer in lowest terms.

(A) $\frac{4x - y}{x} \div \frac{8x - 2y}{3x^2} \cdot \frac{2}{3x}$

(B) $\frac{2x^2}{3} \cdot \frac{6}{x} \div \frac{2x}{5}$

(C) $\frac{x^2 - 8x + 16}{3x} \div \frac{x - 4}{2} \cdot \frac{3x}{4x - 16}$

(D) $\frac{5a}{a^2b^2 - 9} \div \frac{20}{ab} \cdot \frac{3ab + 9}{a^2b}$

(E) $\frac{x^2 - 4}{x} \div \frac{x + 2}{x^2} \cdot \frac{x - 2}{x^3}$

Criterion Test 03-08-07-02

1. Perform the indicated operations and write the answer in lowest terms.

(A) $\frac{3}{x^2} \cdot \frac{x^3}{2} \div \frac{3x}{2}$

(B) $\frac{x^2y^2}{z} \div \frac{z}{x^2y^2} \cdot \frac{2z^2}{x^4y^4}$

(C) $\frac{x^2 - 9}{5} \div \frac{x - 3}{4} \cdot \frac{5}{x + 3}$

(D) $\frac{x^2 + 4x + 4}{3x^3} \cdot \frac{3x}{(x^2 - 4)} \div \frac{x + 2}{x - 2}$

(E) $\frac{x^2}{3x + 4} \cdot \frac{x^2}{3x + 4} \div \frac{x^2}{3x + 4}$

Criterion Test 03-08-07-03

1. Perform the indicated operations and write the answer in lowest terms.

(A) $\frac{x^2}{y^2z^2} \cdot \frac{4}{5} \div \frac{4x^2}{5y^2z^2}$

(B) $\frac{x^2 + 4}{x^2 - 4} \div \frac{2}{3} \cdot \frac{x^2 + 4x + 4}{x + 4}$

(C) $\frac{4x^2 - x + 4}{x - 2x + 1} \div \frac{x - 1}{x + 1} \cdot \frac{6x^2}{2x}$

(D) $\frac{x^2 - x}{x^3} \cdot \frac{x}{x^2 - 1} \div \frac{x^3}{x + 1}$

(E) $\frac{x^3 - x}{x - 1} \div \frac{x + 1}{x} \cdot \frac{x^2}{3}$

Answers to Criterion Tests

Test 03-08-07-01

1. (A) 1 (B) 10 (C) $\frac{1}{2}$

(D) $\frac{3}{4(ab - 3)}$ (E) $\frac{(x - 2)^2}{x^2}$

Test 03-08-07-02

1. (A) 1 (B) 2 (C) 4

(D) $\frac{1}{x^2}$ (E) $\frac{x^2}{3x + 4}$

Test 03-08-07-03

1. (A) 1 (B) $\frac{3(x^2 + 4)(x + 2)}{2(x + 4)(x - 2)}$

(C) $3x$ (D) $\frac{1}{x^4}$ (E) $\frac{x4}{3}$

I. U. #03-08-08

SUMS AND DIFFERENCES

OF

FRACTIONS WITH EQUAL DENOMINATORS

OBJECTIVES:

1. When asked to write the rule for adding and subtracting fractions with equal denominators write,

   "For any real numbers a, b, and c, if $b \neq 0$, then

   $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$

2. Given two fractions with equal denominators, write their sum or difference as a simgle fraction in lowest terms.

ACTIVITIES:

1. Study pages 313 and 314 in SM.
   (Objectives 1, 2)

2. Write enough of the odd numbered part A exercises to be certain that you have met objective number 2. Try some part B and C exercises for more of a challenge.

Criterion Test 03-08-08-01

1. Write the rule for adding and subtracting fractions with equal denominators.

2. Write the sum or difference (as indicated) as a single fraction in lowest terms.

(A) $\frac{4}{17} + \frac{5}{17}$ (B) $\frac{7}{13} - \frac{5}{13}$

(C) $\frac{4}{2x} + \frac{5}{2x} - \frac{7}{2x}$ (D) $\frac{2x + 4}{2} + \frac{3x - 3}{2}$

(E) $\frac{x^2}{x + y} - \frac{y^2}{x + y}$

Criterion Test 03-08-08-02

1. Write the rule for adding and subtracting fractions with equal denominators.

2. Write the sum or difference (as indicated) as a single fraction in lowest terms.

(A) $\frac{2}{7} - \frac{5}{7}$ (B) $\frac{7}{4} + \frac{5}{4}$

(C) $\frac{4x}{2y} - \frac{3x}{2y}$ (D) $\frac{2x}{x + y} - \frac{4x^2}{x + y}$

(E) $\frac{9}{3 - x} - \frac{x^2}{3 - x}$

Criterion Test 03-08-08-03

1. Write the rule for adding and subtracting fractions with equal denominators.

2. Write the sum or difference (as indicated) as a single fraction in lowest terms.

(A) $\frac{3}{7} + \frac{5}{7}$

(B) $\frac{4x}{3} - \frac{3x}{3}$

(C) $\frac{x^2 + 1}{x - 1} - \frac{x^2 - 1}{x - 1}$

(D) $\frac{z^2}{z - x} - \frac{x^2}{z - x}$

(E) $\frac{3ab}{a^2 + b^2} + \frac{a^2 - b^2}{a^2 + b^2}$

Answers to Criterion Tests

Test 03-08-08-01

1. For any real numbers a, b, and c, if $b \neq 0$, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \text{ and } \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

2. (A) $\frac{9}{17}$ (B) $\frac{2}{13}$ (C) $\frac{1}{x}$

(D) $\frac{5x + 1}{2}$ (E) $x - y$

Test 03-08-08-02

1. For any real number a, b, and c, if $b \neq 0$, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \text{ and } \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

2. (A) $-\frac{3}{7}$ (B) 3 (C) $\frac{x}{2y}$

(D) $\frac{2x - 4x^2}{x + y}$ (E) $3 + x$

Answers to Criterion Tests (Cont.)

Test 03-08-08-03

1. For any real numbers a, b, and c, if $b \neq 0$, then $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$

2. (A) $\frac{8}{7}$ or $1\frac{1}{7}$ (B) $\frac{x}{3}$

(C) $\frac{2}{x - 1}$ (D) $z + x$ (E) $\frac{3ab + a^2 - b^2}{a^2 + b^2}$

I. U. #03-08-09

SUMS AND DIFFERENCES

OF

FRACTIONS WITH UNEQUAL DENOMINATORS

You will need to recall how to factor.

OBJECTIVES:

1. Given two fractions with different denominators, rewrite each fraction with their lowest common denominator.

2. Given two fractions with different denominators, write their sum of difference in lowest terms.

ACTIVITIES:

1. Study pages 315 and 316 in SM. (Objectives 1, 2)

2. Do enough of the odd numbered part A, B, and C problems on pages 317 and 318 of SM to be certain that you have met objective two.

Criterion Test 03-08-09-01

1. Rewrite each fraction with the lowest common denominator.

(A) $\frac{1}{3} + \frac{x}{y}$

(B) $\frac{3x - 2}{4} + \frac{2x - 1}{6}$

(C) $\frac{3}{z + 3} - \frac{3}{z - 3}$

(D) $\frac{T}{T^2 - 4} + \frac{3}{T - 2} - \frac{5}{T + 2}$

(E) $\frac{2}{a^2 + ab} - \frac{1}{a + b}$

2. Write the sum or difference (as indicated) in lowest terms.

(A) $\frac{1}{3} + \frac{x}{y}$

(B) $\frac{3x - 2}{4} + \frac{2x - 1}{6}$

(C) $\frac{3}{z + 3} - \frac{1}{z - 3}$

(D) $\frac{T}{T^2 - 4} + \frac{3}{T - 2} - \frac{5}{T + 2}$

(E) $\frac{2}{a^2 + ab} - \frac{1}{a + b}$

Criterion Test 03-08-09-02

1. Rewrite each fraction with the lowest common denominator.

(A) $\frac{1}{4} + \frac{x}{y}$

(B) $\frac{2x - 2}{3} + \frac{2x - 1}{6}$

(C) $\frac{3x}{x + 5} - \frac{x}{x - 5}$

(D) $\frac{3x^2}{x^2 - 9} - \frac{x}{x + 3} + \frac{x}{x - 3}$

(E) $\frac{3}{xy^2 - x} - \frac{5}{y^2 - 1}$

2. Write the sum or difference (as indicated) in lowest terms.

(A) $\frac{1}{4} + \frac{x}{y}$

(B) $\frac{2x - 2}{3} + \frac{2x - 1}{6}$

(C) $\frac{3x}{x + 5} - \frac{x}{x - 5}$

(D) $\frac{3x^2}{x^2 - 9} - \frac{x}{x + 3} + \frac{x}{x - 3}$

(E) $\frac{3}{xy^2 - x} - \frac{5}{y^2 - 1}$

Criterion Test 03-08-09-03

1. Rewrite each fraction with the lowest common denominator.

(A) $\frac{1}{5} + \frac{x}{y}$

(B) $\frac{x^2 - 4}{4} - \frac{x^2 + 4}{3}$

(C) $\frac{x + y}{x + 3} + \frac{x + y}{x - 3}$

(D) $\frac{a}{x^2 - 4} + \frac{b}{x + 2} + \frac{c}{x - 2}$

(E) $\frac{x}{x^3 - x} + \frac{x}{x^2 + x}$

2. Write the sum or difference (as indicated) in lowest terms.

(A) $\frac{1}{5} + \frac{x}{y}$

(B) $\frac{x^2 - 4}{4} - \frac{x^2 + 4}{3}$

(C) $\frac{x + y}{x + 3} + \frac{x + y}{x - 3}$

(D) $\frac{a}{x^2 - 4} + \frac{b}{x + 2} + \frac{c}{x - 2}$

(E) $\frac{x}{x^3 - x} + \frac{x}{x^2 + x}$

Answers to Criterion Tests

Test 03-08-09-01

1. (A) $\frac{y}{3y} + \frac{3x}{3y}$

(B) $\frac{3(3x - 2)}{12} + \frac{2(2x - 1)}{12}$

(C) $\frac{(z - 3)3}{z^2 - 9} - \frac{(z + 3)3}{z^2 - 9}$

(D) $\frac{T}{T^2 - 4} + \frac{(T + 2)3}{T^2 - 4} - \frac{(T - 2)5}{T^2 - 4}$

(E) $\frac{2}{a^2 + ab} - \frac{a}{a^2 + ab}$

2. (A) $\frac{y + 3x}{3y}$

(B) $\frac{13x - 8}{12}$

(C) $\frac{2z - 12}{z - 9}$

(D) $\frac{16 - T}{T^2 - 4}$

(E) $\frac{2 - a}{a^2 + ab}$

Answers to Criterion Tests (Cont.)

Test 03-08-09-02

1. (A) $\frac{y}{4y} + \frac{4x}{4y}$

(B) $\frac{2(2x - 2)}{6} + \frac{2x - 1}{6}$

(C) $\frac{(x - 5)3x}{x^2 - 25} - \frac{(x + 5)x}{x^2 - 25}$

(D) $\frac{3x^2}{x^2 - 9} - \frac{(x - 3)x}{x^2 - 9} + \frac{(x + 3)x}{x^2 - 9}$

(E) $\frac{3}{xy^2 - x} - \frac{5x}{xy^2 - x}$

2. (A) $\frac{y + 4x}{4y}$

(B) $\frac{6x - 5}{6}$

(C) $\frac{2x^2 - 20x}{x^2 - 25}$

(D) $\frac{3x^2 + 6x}{x^2 - 9}$

(E) $\frac{3 - 5x}{xy^2 - x}$

Answers to Criterion Tests (Cont.)

1. (A) $\frac{y}{5y} + \frac{5x}{5y}$ (B) $\frac{3(x^2 - 4)}{12} - \frac{4(x^2 + 4)}{12}$

(C) $\frac{(x - 3)(x + y)}{x^2 - 9} + \frac{(x + 3)(x + y)}{x^2 - 9}$

(B) $\frac{a}{x^2 - 4} + \frac{(x - 2)b}{x^2 - 4} + \frac{(x + 2)c}{x^2 - 4}$

(E) $\frac{x}{x^3 - x} + \frac{(x - 1)x}{x^3 - x}$

2. (A) $\frac{y + 5x}{5y}$ (B) $\frac{-x^2 - 28}{12}$

(C) $\frac{2x^2 + 2xy}{x^2 - 9}$ (D) $\frac{a + bx - 2b + cx + 2c}{x^2 - 4}$

(E) $\frac{x}{x^2 - 1}$

I. U. #03-08-10

MIXED EXPRESSIONS

OBJECTIVES:

1. Given a mixed expression, write it as a fraction in lowest terms and conversely.

ACTIVITIES:

1. Study page 318, SM, and write the odd numbered part A exercises on page 319.
   (Objective 1)

2. Review the division algorithm on page 235 if necessary.
   (Objective 1)

Criterion Test 03-08-10-01

1. Write as a fraction in lowest terms.

(A) $x + \frac{3x}{x + y}$ (B) $x + 3 + \frac{1}{x + 3}$

(C) $3 + \frac{a - b}{a + b}$

2. Write as a mixed expression.

(A) $\frac{188}{15}$ (B) $\frac{12 - 6x^2}{2x}$ (C) $\frac{2x^2 - x - 4}{2x - 3}$

Criterion Test 03-08-10-02

1. Write as a fraction in lowest terms.

(A) $x + \frac{y}{x + y}$ (B) $2x - y + \frac{1}{x + 1}$

(C) $3x - 2 - \frac{x + 5}{x - 2}$

2. Write as a mixed expression.

(A) $\frac{300}{14}$ (B) $\frac{10 + 15b^2}{5b}$ (C) $\frac{2x^2 - x - 4}{x + 1}$

Criterion Test 03-08-10-03

1. Write as a fraction in lowest terms.

(A) $a + \frac{b}{a - b}$ (B) $x + 3 + \frac{3}{3 + x}$

(C) $x + 2 - \frac{x + 1}{x - 1}$

2. Write as a mixed expression.

(A) $\frac{300}{21}$ (B) $\frac{3 - 12x^2}{3x}$ (C) $\frac{6x^2 - 7x + 4}{2x - 1}$

Answers to Criterion Tests

Test 03-08-10-01

1. (A) $\frac{x^2 + xy + 3x}{x + y}$ (B) $\frac{x^2 + 6x + 10}{x + 3}$

   (C) $\frac{4a + 2b}{a + b}$

2. (A) $12\frac{8}{15}$ (B) $-3x + \frac{6}{x}$ (C) $x + 1 + \frac{-1}{2x - 3}$

Test 03-08-10-02

1. (A) $\frac{x^2 + xy + y}{x + y}$ (B) $\frac{2x^2 + 2x - xy - y + 1}{x + 1}$

   (C) $\frac{3x^2 - 9x - 1}{x - 2}$

2. (A) $21\frac{3}{7}$ (B) $3b + \frac{2}{b}$ (C) $2x - 3 - \frac{1}{x + 1}$

Test 03-08-10-03

1. (A) $\frac{a^2 - ab + b}{a - b}$ (B) $\frac{x^2 + 6x + 12}{3 + x}$

   (C) $\frac{x^2 - 3}{x - 1}$

2. (A) $14\frac{6}{21}$ (B) $-4x + \frac{1}{x}$

   (C) $3x - 2 + \frac{2}{2x - 1}$

I. U. #03-08-11

COMPLEX FRACTIONS

OBJECTIVES:

1. Given a complex fraction, simplify it.

ACTIVITIES:

1. Study page 320, SM, until you understand both methods of changing complex fractions to simple ones. (Objective 1)

2. Do enough of the odd numbered part A exercises on page 321 to be certain that you have met objective 1.

3. Try some of the part B and C exercises to see if you can simplify more complex complex fractions.

Criterion Test 03-08-11-01

1. Simplify.

(A) $\dfrac{\frac{2}{3}}{\frac{3}{4}}$

(B) $\dfrac{\frac{a}{b}}{\frac{a}{c}}$

(C) $\dfrac{\frac{x^2 - 9}{x}}{x - 3}$

(D) $\dfrac{\frac{7}{8} + \frac{5}{6}}{\frac{5}{12} - 2}$

(E) $\dfrac{\frac{x}{y} - 2}{\frac{x}{y} + 1}$

Criterion Test 03-08-11-02

1. Simplify

(A) $\dfrac{\frac{5}{8}}{\frac{2}{3}}$

(B) $\dfrac{\frac{a}{b}}{\frac{2a}{3b}}$

(C) $\dfrac{\frac{x^2 - 16}{4}}{x + 4}$

(D) $\dfrac{\frac{x}{y} + 2}{1 - \frac{x}{y}}$

(E) $\dfrac{\frac{x}{x + y} + 2}{\frac{x}{x + y} - 2}$

Criterion Test 03-08-11-03

1. Simplify:

(A) $\dfrac{\frac{3}{2}}{\frac{2}{3}}$

(B) $\dfrac{\frac{a^2b^2}{c^2}}{\frac{ab^2}{c^3}}$

(C) $\dfrac{\frac{2}{3} + 3}{\frac{3}{5} + 6}$

(D) $\dfrac{x - \frac{5}{x + 2}}{x + \frac{5}{x + 2}}$

(E) $\dfrac{\frac{2x + 1}{x - 1} + \frac{x}{x + 1}}{\frac{5}{x^2 - 1}}$

Answers to Criterion Tests

Test 03-08-11-01

1. (A) $\frac{8}{9}$ (B) $\frac{c}{b}$ (C) $\frac{x + 3}{x}$ (D) $-\frac{41}{38}$

(E) $\frac{x - 2y}{x + y}$

Test 03-08-11-02

1. (A) $\frac{15}{16}$ (B) $\frac{3}{2}$ (C) $\frac{x - 4}{4}$ (D) $\frac{x + 2y}{y - x}$

(E) $-\frac{3x + 2y}{x + 2y}$

Test 03-08-11-03

1. (A) $\frac{9}{4}$ (B) ac (C) $\frac{5}{9}$ (D) $\frac{x^2 + 2x - 5}{x^2 + 2x + 5}$

(E) $\frac{3x^2 + 2x + 1}{5}$

THE END
OF
PACKAGE 03-08